HYDRAULIC CHARACTERISTICS OF PADDLE IMPELLERS WITH FLAT INCLINED BLADES* **

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A model is presented for the derivation of hydraulic characteristics of paddle impellers with flat inclined blades: the pumping capacity of the impeller, the drag force on the blade and the power input. The model proposed is based on description of pattern of the mean velocity field in the stream leaving the blades of the investigated impellers. The model velocity profile is described by means of a single parameter which must be estimated from experimental results. The experimentally obtained quantity was the time of primary circulation of a tracer particle in the mixed charge which in turn served to calculate the pumping capacity of the mixer. The effects were examined of the physical and geometrical properties of the mixed system on the mentioned parameter of the velocity field under the turbulent regime of the charge. The most marked effect detected was that of the inclination of the blades and of the number of the blades for all investigated types of the impellers on all the three selected characteristics. The relative size and position of the impeller within the system under consideration (a cylindrical vessel with radial baffles) do not affect the mentioned characteristics; all the three hydraulic characteristics are independent of the Reynolds number within the investigated range of physical properties.

Hydraulic characteristics of the rotating impellers depend mostly on the form of the velocity profile in the stream leaving from the blades of the impeller. The velocity profile in the system in question has been subject of many experimental and theoretical studies. All studies on paddle impellers with vertical flat blades, *e.g.* cit.[1-8], as well as those with inclined flat blades, *e.g.* cit.[9-12], report the mean velocity profile in systems with radial baffles to be similar to that existing in systems without baffles and consists of two subregions: the inner, linear and the outer, hyperbolic one (Fig. 1). While in the region of the former one the mixed liquid behaves ideally, the finite viscosity of liquid in the region of the latter one becomes clearly manifest. This description corresponds to the so called Rankin vortex consisting of a vortex core

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and external potential vortex. The limits between the two vortices, a circle of radius r_c, is then a locus of the points of maximum velocity on the examined velocity profile. The radius of this limit is then a function of the geometry of the blade (inclination and eventually the number of the blades), the regime of the flow past the blade (turbulent or laminar) and the presence of baffles within the mixing vessel; these functions must be found experimentally.

Theoretical and experimental studies of pumping capacity of the paddle impellers with inclined flat blades are numerous[9,10,13-16]. These studies indicate that the above characteristics of such impellers significantly affect the value of the pumping capacity. To a lesser extent affect this quantity also the relative size of the impeller and the position of the impeller in the system. As long as the regime of the flow within the charge remains turbulent one can neglect with sufficient accuracy the effect of the Reynolds number. The drag on the blade has been investigated experimentally and theoretically only in case of flat vertical blades[6,17]. From these papers it can be concluded mainly that without the knowledge of the course of the tangential component of the mean velocity of liquid along the blade the drag force or eventually the pressure distribution over the surface of the blade cannot be calculated. These papers further indicate an unambiguous relation between the drag force and the power input of the impeller. In the established relation one can clearly distiguish the effect of both the velocity profile and the blade geometry. Experimental studies of the power input on the paddle impellers with flat blades have been carried out both with the inclined and the vertical blades[6,17-21]. These results suggest a significant effect of the shape, inclination angle and the number of the blades on the power input of the impeller. On the other hand, under the turbulent regime of the charge the Reynolds number plays no role.

THEORETICAL

Consider a mixed system consisting of a cylindrical vessel with flat bottom and four radial baffles reaching the bottom. The vessel is filled with a newtonian liquid up to a liquid height H at rest equal the inner diameter of the vessel, D. A paddle impeller with inclined flat blades pumping the liquid downward rotates in the axis of the vessel. The diameter of the impeller, d, and its height above the bottom, H_2, do not exceed one half of the inner vessel diameter. Let us introduce the following simplifying assumptions for the above system:

I) The flow of the mixed charge is a quasistationary turbulent flow.

II) The flow within the hypothetical region of the rotating impeller (the so called rotor region) is axially symmetric with respect to the axis of the rotor shaft.

III) The effect of the hub of the impeller on the flow within the rotor region may be neglected. The origin of the chosen frame of reference is located at the point of intersection of the symmetry axis of the shaft and the lower edge of the rotating blades.

IV) On the basis of the experimental investigation of the flow within the stream leaving from the rotor region[9,10,14] the radial profile of the mean velocity within this stream may be described by the following equations (Fig. 1 and 2). For the axial component of the mean velocity

$$w_{ax}(r) = 2\pi nrk\,, \quad [r \leqq r_c]\,, \tag{1a}$$

$$w_{ax}(r) = C_{ax}/r\,, \quad [r \geqq r_c] \tag{1b}$$

and for the tangetial component

$$\bar{w}_{tg}(r) = 2\pi nr\,, \quad [r \leqq r_c]\,, \tag{2a}$$

$$\bar{w}_{tg}(r) = C_{tg}/r\,, \quad [r \geqq r_c]\,. \tag{2b}$$

The constants C_{ax} and C_{tg} in Eqs (*1b*) and (*2b*) may be expressed by means of the parameter r_c (the limits of the subregions of the velocity profile) from the relations

$$C_{ax} = 2\pi nr_c^2k\,, \quad C_{tg} = 2\pi nr_c^2\,. \qquad (3a), (3b)$$

The magnitude of the parameter k in Eqs (*1a*) and (*3a*) follows from the geometry of the flow past the inclined blade of the impeller making an angle α with the horizontal plane (Fig. 3). For this parameter we have

$$k = \operatorname{cotg} \alpha\,. \tag{4}$$

The equations for the velocity profile in the stream leaving the rotor region characterize the two adjacent subregions: The inner one, where the liquid rotates at the

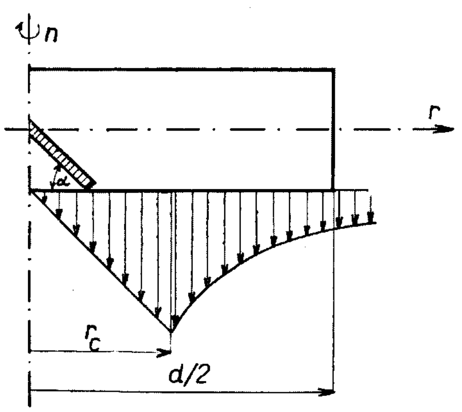

FIG. 1

Velocity Profile in the Stream of Liquid Leaving the Rotor Region of a Paddle Impeller with Flat Inclined Blades

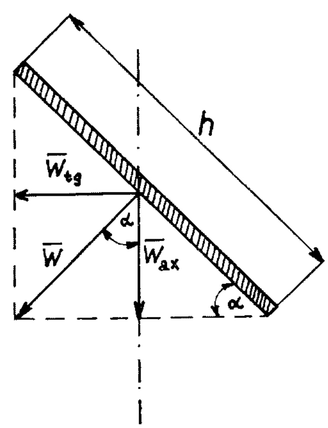

FIG. 2

Transformation of Velocity of Liquid Departing from a Flat Inclined Blade speed identical with the number of revolution of the blades of the impeller, and the outer subregion, where, due to the effect of viscous and inertial forces, the liquid boundary layer departs from the surface of the blades and the velocity of the flowing liquid thus differs from that of the blade.

Radial coordinate, r_c, of the origin of the hyperbolic profile of the mean velocity (Eq. (*1b*) and (*2b*)) depends on the width, inclination and the number of the impeller blades and also on the size and position of the impeller within the mixed vessel. The outer limits of the latter subregion does not exceed the radius of the impeller, $d/2$.

The pumping capacity of the paddle impellers with inclined flat blades is defined as a volumetric flow rate of liquid through a unit area of an orthogonal projection of the circle made by the rotating blades into the plane passing through the hub of the impeller perpendicularly to the axis of rotation. This quantity can be obtained from the known radial profile of axial component of the mean velocity $\bar{w}_{ax} = \bar{w}_{ax}(r)$ in the stream leaving from the rotor region using the assumptions *II* and *IV* as

$$\dot{V} = 2\pi \int_0^{d/2} \bar{w}_{ax}(r)\, r\mathrm{d}r\,, \tag{5}$$

The integration after substitution from Eq. (*3a*) yields

$$\dot{V} = 4\pi^2 nk[(d/2)\, r_c^2 - (2/3)\, r_c^3]\,. \tag{6}$$

Introducing a new dimensionless parameter

$$c = 2r_c/d \tag{7}$$

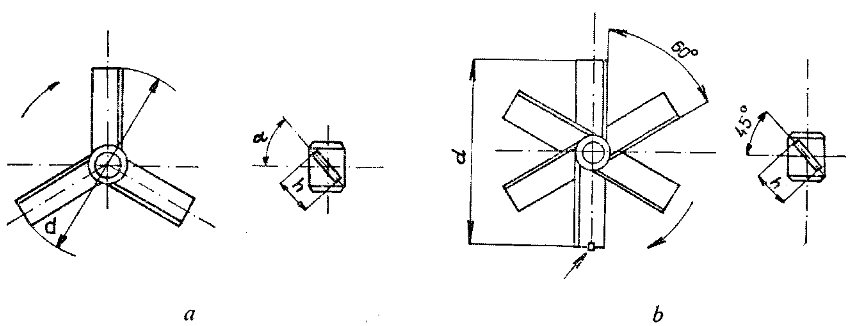

FIG. 3

Paddle Impellers with Inclined Flat Blades:

a) Three-blade impeller, ($h = 0{\cdot}2$ d); *b*) Six-blade impeller, $\alpha = 45°$ ($h = 0{\cdot}2$ d).

the value of which is confined between zero and unity, Eq. (6) can be expressed conveniently in a dimensionless form as

$$\mathrm{K_P} = \dot{V}/nd^3 = (\pi^2 kc^2/2)(1 - 2c/3), \tag{8}$$

where $\mathrm{K_P}$ represents the so called flow rate criterion. This equation indicates that the pumping capacity of the impellers under consideration depends on one hand on the inclination of the blades, and, on the other hand, on position of the limits between the two subranges of the velocity profile, $\bar{w}_{ax} = \bar{w}_{ax}(r)$, characterized by the constant c. The constant c itself, of course, depends both on the inclination and the number of the impeller blades[15].

The drag force of the impeller blade is given by the equation[22,23]

$$f_D = \iint_A \xi_D \, 1/2 \, \varrho \bar{w}^2_{tg,rel} (A_D) \, dA_D \, . \tag{9}$$

In view of the assumption *II* the elementary surface dA_D, perpendicular to the motion of the blade, may be expressed as

$$dA_D = h \sin \alpha \, dr \, , \tag{10}$$

which indicates that the only independent variable in Eq. (9) is the radius r. The surface integral over the area A_D thus transforms into a simple integral over the radius of the blade $d/2$. The quantity h (Fig. 2) is the width of the blade and the product $h\sin\alpha$ its projection into the direction perpendicular to the roration of the blade. In view of the assumption *I* the value of the local drag coefficient of resistance ξ_D may be regarded as independent of the position of the impeller as well as the relative tangetial component of the mean velocity of liquid with respect to the blade, $\bar{w}_{tg,rel}$. The dependence on the shape of the blade, of course, is anticipated. The quantity $\bar{w}_{tg,rel}$ equals*

$$\bar{w}_{tg,rel} = 2\pi nr - 2\pi nr = 0 \, , \quad [r \leqq r_c] \, , \tag{11a}$$

$$\bar{w}_{tg,rel} = 2\pi n(r^2 - r_c^2)/r \, , \qquad [r \in \langle r_c; d/2 \rangle] \, . \tag{11b}$$

The product $2\pi nr$ in the set (11) defines the tangetial (peripheral) velocity of the given point of the blade of the coordinate r. From the above set of equations it follows that in the subregion of radial coordinate $r \in \langle 0; r_c \rangle$ the blade (or perhaps, better, the

* In fact, the sign of the relative velocity of the liquid should be negative. However, as we are concerned with the drag force on the blade offered against the flowing liquid the quantity f_D is taken positively.

liquid) offers no resistance to the liquid (the blade). That is concentrated in the other subrange of the profile and its course is characterized by the pressure acting on the blade

$$\mathrm{d}f_\mathrm{D}/\mathrm{d}A_\mathrm{D} = \xi_\mathrm{D}\, 1/2\, \varrho \bar{w}^2_{\mathrm{tg,rel}}\,. \tag{12}$$

This pressure on the blade ranges in the following limits

$$\mathrm{d}f_\mathrm{D}/\mathrm{d}A_\mathrm{D} = 0\,, \quad [r \leqq r_\mathrm{c}]\,, \tag{13a}$$

$$\mathrm{d}f_\mathrm{D}/d_\mathrm{AD} = 2\pi^2\, \xi_\mathrm{D}\varrho n^2(r^2 - r_\mathrm{c}^2)^2/r^2\,, \quad [r \in \langle r_\mathrm{c};\, d/2\rangle]\,. \tag{13b}$$

The discussed quantity reaches a maximum at the point $d/2$ where we have

$$\mathrm{d}f_\mathrm{D}(r = d/2)/\mathrm{d}A_\mathrm{D} = \xi_\mathrm{D}\varrho[(\pi \mathrm{dn})^2/2]\,(1 - c^2)^2\,. \tag{14}$$

The drag force on the blade is obtained by substituting from Eqs (*10*) and (*11b*) into Eq. (*9*), integrating and substituting for the quantity c from Eq. (*7*). Thus we have

$$f_\mathrm{D} = (\sin\alpha)\,(\pi^2/2)\,\xi_\mathrm{D}\varrho h n^2 d^3[(1/6)\,(1 + 8c^3) - c^2\,(1 - c^2/4)]\,. \tag{15}$$

The drag force for an impeller with N blades, if N is such that no interference occurs, may be computed as a sum of the drag forces of individual blades. In the dimensionless form namely

$$F_\mathrm{tgM} = Nf_\mathrm{D}/\varrho n^2 d^4 = \mathrm{N}\,(\sin\alpha)\,(\pi^2/2)\,\xi_\mathrm{D}(h/d)\,[(1/6)\,(1 + 8\,c^3) - c^2(1 - c^2/4)]\,. \tag{16}$$

The power input of the impeller can be computed at constant frequency of revolution

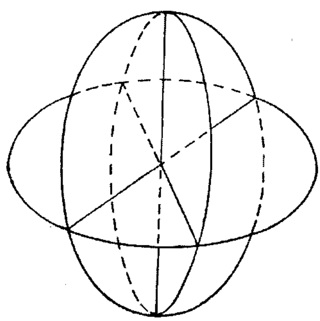

FIG. 4
Tracer (Indicating) Particle

from the following equation[22]

$$P = 2\pi n N \int_0^{d/2} r \, \mathrm{d}f_{\mathrm{D}} \,. \tag{17}$$

Eq. (*17*) already takes into account the assumption *II*, *i.e.* the axial symmetry of the stream leaving from the rotor region. Substituting from Eqs (*9*), (*11*) into (*17*) one obtains

$$P = 4\pi^3 N \sin\alpha \, \xi_{\mathrm{D}} \varrho h n^3 \int_{r_c}^{d/2} (r^3 + r_c^4/r - 2r_c^2 r) \, \mathrm{d}\, r \,. \tag{18}$$

Integrating Eq. (*18*) and substituting for c from Eq. (*7*) we finally arrive at the resulting dimensionless equation for the Euler (power input) number for the impeller

$$\begin{gathered} \mathrm{Eu_M} = P/\varrho n^3 d^5 = N(\sin\alpha)(\pi^3/4)\,\xi_{\mathrm{D}}(h/d)\,. \\ [(1 - c^4)/4 + c^4 \ln(1/c) - c^2(1 - c^2)]\,. \end{gathered} \tag{19}$$

The theoretically obtained equations for all discussed hydraulic characteristics of paddle impellers with inclined flat blades are thus closely associated with the profile of the mean velocity of liquid emerging from the rotor region.

EXPERIMENTAL

Description of the dependence of the hydraulic characteristics of paddle impellers on the velocity profile require experimental determination of the parameter c defined by Eq. (*7*). This quantity may be computed from the known flow rate criterion, K_P — see Eq. (*8*), calculated from the experimentally obtained value of the pumping capacity of the impeller. Thus in view of the above discussed effects on the parameter c the following function must be investigated experimentally

$$c = c(\mathrm{Re_M}, \sin\alpha, \mathrm{N}, D/d, D/H_2)\,. \tag{20}$$

The measurements were carried out in vessels with flat bottom provided with four radial baffles of the width $b = 0{\cdot}1D$, where the inner diameter of the vessel, D, equals 290 and 190 mm. The charges used were distilled water ($\eta/\varrho = 1{\cdot}0 \,.\, 10^{-6} \mathrm{m^2/s}$) and aqueous solution of glycerol ($\eta/\varrho = 2{\cdot}34 \,.\, 10^{-6}$). The liquid height at rest, H, reached D. The vessels were placed in a thermostat kept at 20°C. The measurements were carried out with three-bladed impellers with flat blades inclined at an angle $\alpha = 24$, 35 and 45°, and with N-bladed impellers ($N = 2, 3, 4, 5, 6$) with flat blades inclined at $\alpha = 45°$ (Fig. 3*a*, *b*). The impellers rotated in such a direction so as to pump the liquid toward the bottom. The impeller was driven by a DC electromotor 0·5 kW geared to permit a reduction of the frequency of revolution by as much as a factor of four. The frequency of revolution was kept constant to within $\pm 1\%$ by means of a magnetic regulator.

In order to determine the pumping capacity of the impellers a circulation model of the flow of the charge was used[9,24]. In the experimentally verified properties of such a system[9,10,15] the pumping capacity of the mixer may be computed from

$$\dot{V} = V/\bar{\tau}_P\,, \tag{21}$$

where V is the total volume of the mixed charge. For $D = 190$ mm this volume equalled $5{\cdot}41\,.\,10^{-3}\text{m}^3$; for $D = 290$ mm $V = 19{\cdot}15\,.\,10^{-3}\text{m}^3$. Quantity $\bar{\tau}_P$ is the mean time of primary circulation — the mean time interval between two consecutive passages of a check element of liquid through the rotor region. The check element can be modelled suitably by a tracer particle ensuring that its motion follows that of the liquid itself. The average time of circulation, $\bar{\tau}_P$, was observed during experiments visually. The total time of m_P individual passages, τ_P, was measured by a stop watch and m_P was recorded by means of an electromagnetic counter. The mean time of primary circulation time was then assessed from

$$\bar{\tau}_P = \tau_P/m_P\,. \tag{22}$$

600 passages of the tracer particle through the rotor region were recorded in a single run for a given physical and geometry conditions. The run was divided into six parts of 100 passages in each of them. The used tracer particle[25] (Fig. 4) was of spherical outline and consisted of three discs 6 mm in diameter and 0·2 mm thick. The particle was made of PVC (density 1100 kg/m^3). The design of the tracer particle ensures that it moves within the stream as the liquid displaced by its volume. The resistance to flow past such a body is large and the difference in inertia in comparison with the liquid is small. Consequently, the indication of the motion is satisfactory.

In the experiments we took as independent variables the following quantities: n, η/ϱ, d, α, N and the distance of the lower edge of the impeller blades, H_2, from the bottom. The frequency of impeller revolution was held constant to $\pm 1\%$. The temperature of the charge was held constant at $20 \pm 0{\cdot}5$°C which caused fluctuation of the kinematic viscosity within 1·5%. The diameter of the impeller was measured with the accuracy of $\pm 0{\cdot}2$ mm, the angle of inclination $\pm 0{\cdot}5$ deg. The distance from the bottom to $\pm 0{\cdot}5$ mm.

As the dependent variable we took the primary time of circulation regarded as a random variable the accuracy of which depends on the number of repeated experiments. In several randomly selected experiments it was tested whether the number of passages of the tracer particle through the rotor region at the given experiment enables the required accuracy of measurement given by the standard mean square deviation of the of time primary circulation to be achieved. The required relative value of the mean square deviation was 6% .In all tests the error found was better or at least at the required level.

Summarily it can be concluded that the values of independent variables carried practically negligible errors in comparison with that of the dependent variable.

RESULTS AND DISCUSSION

The values of the pumping capacity of the mixer, $\dot{V}$, were obtained by substituting the found mean of time primary circulation, $\bar{\tau}_P$, obtained from the measured values of τ_P and m_P from Eq. (22). The volume of the mixed charge for a given size of the vessel was constant. The pumping capacity for the given value of the diameter d and the frequency of revolution of the impeller was recalculated to the value of the flow rate criterion, K_P. The Newton iteration with the maximum error of the last iteration equalling $1\,.\,10^{-5}$ was used for calculation of the parameter c from Eq. (8). The dependent and independent variables for further processing were expressed dimensionless.

The effect of the Reynolds number on the value of the parameter c of the velocity profile was assessed from the analysis of the following dependence

$$c = C\,\mathrm{Re}_{\mathrm{M}}^{\varkappa}(D/d)^{\varepsilon}\,(D/H_2)^{\beta}\,, \quad [\alpha = \mathrm{const}]\,. \tag{23}$$

The parameters C, $\varkappa$, ε, and β for a given type of impeller were determined by the least-square non-linear regression. In view of the earlier presented results of the effect of the Reynolds number on the pumping capacity and the Euler number for the examined types of impellers under the turbulent regime of the flow of the charge the estimate of the standard deviation, $\sigma_{\varkappa}$, of the exponent of the latter group was used to ascertain whether the value of $\varkappa$ is statistically significant. This was achieved by testing the hypothesis whether the value of the exponent equals zero[26]. For all examined arrangements this hypothesis on the 95% significance level could be accepted. For this reason the effect of the Reynolds number on c may be neglected as long as the regime of the flow of the charge remains turbulent, *i.e.* if $\mathrm{Re}_{\mathrm{M}} > 1\,.\,10^4$.

The effect of the geometry of the impeller and the mixed system on the velocity profile within the stream of liquid leaving the blades of the impeller with inclined flat blades was tested, in view of the already found effect of the Reynolds number on c, by analysis of the following relations

impeller in Fig. 3*b*:

$$c = A_{11}N^{a_{11}}\,, \quad [\alpha = 45°,\, D/d = D/H_2 = 30]\,; \tag{24}$$

impeller in Fig. 3*a*:

$$c = A_{21}(\sin\alpha)^{c_{21}}\,(D/H_2)^{b_{21}}\,, \quad [N = 3,\, D/d = 3]\,, \tag{25a}$$

$$c = A_{22}(\sin\alpha)^{c_{22}}\,(D/H_2)^{b_{22}}, \quad [N = 3,\, D/d = 4]\,, \tag{25b}$$

$$c = A_{23}(\sin\alpha)^{c_{23}}\,(D/H_2)^{b_{23}}\,, \quad [N = 3,\, D/d = 5] \tag{25c}$$

and finally by generalization of Eqs (*25a*)–(*25c*) in the form

$$c = A_{31}(\sin\alpha)^{c_{31}}\,(D/H_2)^{b_{31}}\,(D/d)^{a_{31}}\,, \quad [N = 3]\,. \tag{26}$$

The exponents and the constants in the power expressions (*24*)–(*26*) were obtained by the least-square method using multiple non-linear regression. Apart from the estimates of the coefficients and exponents for individual independent variables the computational method was also used to obtain the estimates of the standard deviations of these coefficients and exponents (Table I).

The effect of the geometry of the system and the shape of the blades on the flow pattern within the liquid leaving from the rotor region of an impeller with flat inclined blades. The effects examined were those of the relative size of the impeller and vessel D/d and position of the impellers within the system characterized by the simplex D/H_2. Both geometrical simplexes have a statistically significant effect on the value of the parameter of the investigated velocity profile, c: The greater the relative size of the impellers and the farther relatively from the bottom of the vessel, the higher the value of the parameter c, *i.e.* the maximum velocity in the stream emerging from the rotor region increases with increasing value of the above two simplexes. These two effects, though, are not very conspicious (particularly not that of D/d) which is in agreement with the earlier presented experimental results regarding the flow within the region under consideration[9,13]. As far as the effect of D/H_2 on c is concerned it can be concluded that decreasing distance between the impeller and the bottom affects primarily the field of streamlines in the stream leaving the blades of the rotating impeller and hence the profile of the mean velocity in this stream.

The shape of the impeller, characterized in the experiments by the angle of inclination, α, of the blades from the horizontal level and the number of the blades, N, affects considerably the found velocity profile. With increasing number of the blades, as well as the inclination angle, the value of the parameter c considerably increases. This indicates that the position of the maximum of the velocity profile shifts toward the tip of the blades. Thus in the range covered by experiments, *i.e.* up to the inclination of the blades equal to 45°, the subregion of the vortex core (where the liquid behaves ideally) expands if the angle of inclination (made with the horizontal level) increases. For impellers with a smaller value of the angle alpha, or for impellers with a smaller number of the blades, the departure of the boundary layer from the blade occurs apparently at positions characterized by a smaller radial distance r (Fig. 1) than in case of impellers with large alpha and/or N.* The subregion of the outer potential vortex (characterized by the profile of the mean velocity (*1b*) or (*2b*)), where the corresponding velocity component differs from that of the blade, then expands toward the axis of the impeller. The limits between the two subregions thus shifts toward lower values of the radial coordinate.

In order to achieve greater pumping effects of the paddle impellers with inclined flat blades it is useful, within the investigated ranges of both the angle of inclination as well as the number of the blades, to use maximum values. The effect of the number

* This statement can be supported by the results of experimental investigation of erosion of the blade of the given type of impeller in a heterogeneous solid–liquid charge under the conditions identical to those of this work[27]. With decreasing value of alpha as well as the number of the blades the intensity of erosion in the subregion of the blade, where the boundary layer has been separated from the blade, was considerably intensified owing to the different mean velocity of the charge and the number of revolution of the blade.

of the blades on the flow criterion can be expressed by the following linear expression

$$\mathrm{K_P} = QN + P\,, \quad [N \in \langle 2; 6\rangle; D/d = 3; \mathrm{Re_M} > 1{\cdot}0 : 10^4]\,, \tag{27}$$

where

$$Q = 0{\cdot}048;\ \sigma_\mathrm{Q} = 0{\cdot}0018 \quad \text{and} \quad P = 0{\cdot}659;\ \sigma_\mathrm{P} = 0{\cdot}0076.$$

This linear dependence of the pumping capacity of the impellers on the number of the blades is in agreement with the theory for mutual interaction of the fluid and the rotor of a pump or an airplane propeller[28]. It may thus be concluded that within the investigated range of the number of the blades no mutual force interaction occurs.

The effect of the Reynolds number on the magnitude of the parameter c of the investigated profile of the mean velocity was found statistically insignificant. This fact confirms also the validity of the assumption *I* regarding the flow past the rotating impeller and it is also in agreement with the results of other experimental studies, *e.g.*[6,10,13,15,16,18,19]: as long as the value of the Reynolds number does not drop below 10^4 the character of the flow may be regarded as turbulent – the automodel one. It should be noted that the established independence of the parameter c on the variable $\mathrm{Re_M}$ implicitely involves that the assumption *II* is satisfied. If for some of the lower frequencies of revolution of the impeller the axial symmetry of the stream leaving from the rotor region under consideration was violated, the parameter c would display certain dependence on the Reynolds number indicating also that the assumption *II* was not met.

The force action between the blades of the impeller and the flowing liquid. In the theoretical part we have derived relationships for the force (or pressure) acting between the blade and the liquid streaming past it enabling its calculation from the known profile of the mean velocity at the exit from the rotor region. The experimental

TABLE I

The Effect of Geometry of Mixed System and the Shape of Blades on the Velocity Profile in the Stream Leaving the Rotor Region of an Impeller with Flat Inclined Blades ($\mathrm{Re_M} > 10^4$).

A_{ij}	$\sigma_{A_{ij}}$	a_{ij}	$\sigma_{a_{ij}}$	b_{ij}	$\sigma_{b_{ij}}$	c_{ij}	$\sigma_{c_{ij}}$	i	j	Equation
0·425	0·0072	0·139	0·012	—	—	—	—	1	1	*(24)*
0·767	0·038	—	—	0·120	0·044	1·363	0·054	2	1	*(25a)*
0·745	0·030	—	—	0·119	0·027	1·505	0·030	2	2	*(25b)*
0·721	0·042	—	—	0·122	0·038	1·461	0·042	2	3	*(25c)*
0·669	0·017	0·057	0·017	0·120	0·017	1·442	0·022	3	1	*(26)*

results indicate that the inclination of the blades as well as the number of the blades considerably contribute to the magnitude of the two quantities. Quantitative expression of this effect, however, is rather complicated as the two quantities appear to be complex functions of the parameter c (see Eq. (*14*) and (*16*)). Moreover, it can be expected that the drag coefficient, ξ_D, depends also on the angle of inclination of the blades alpha. From the results of the regression analysis given in Eqs (*24*)–(*26*) and the values of their parameters (Table I) we have obtained the following functions of the parameter c appearing in the mentioned equations for the angle alpha ranging between 24° and 45° and the number of the blades ranging in the interval $N \in \langle 2; 6 \rangle$. These functions are

$$\varphi_1(c) = (1 - c^2)^2\,, \quad [\text{Eq. } (14)]\,, \tag{28a}$$

$$\varphi_2(c) = 1/6(1 + 8c^3) - c^2(1 - c^2/4)\,, \quad [\text{Eq. } (16)]\,, \tag{28b}$$

$$\varphi_3(c) = (1 - c^4)/4 + c^4 \ln(1/c) - c^2(1 - c^2)\,, \quad [\text{Eq. } (19)]\,. \tag{28c}$$

The results of the correlation in the form (*26*) then yielded for the given range of the angle alpha the following proportionalities

$$\varphi_1(c) \sim (\sin\alpha)^{-1.0}\,, \tag{29a}$$

$$\varphi_2(c) \sim (\sin\alpha)^{-0.74}\,, \tag{29b}$$

$$\varphi_3(c) \sim (\sin\alpha)^{-1.3}\,. \tag{29c}$$

The correlation in the form (*24*) then yielded for the given range of number of the blades N the proportionality

$$\varphi_3(c) \sim N^{-0.32}\,. \tag{30}$$

Kvasnička reports[6] the following correlation for the Euler number and the given type of paddle impellers (Fig. 3*a*)

$$\mathrm{Eu_M} = 1{\cdot}672\,(\sin\alpha)^{1.74}\,, \quad [N = 3,\ \mathrm{Re_M} > 1{\cdot}0\,.\,10^4,\ \alpha \in \langle 10°; 45° \rangle\,. \tag{31}$$

On combining Eqs (*19*), (*29c*) and (*31*) one arrives at the following proportionality

$$\xi_D \sim (\sin\alpha)^{2.04}\,, \tag{32}$$

thus we have

$$\mathrm{d}\,f_D(r = d/2)/\mathrm{d}\,A_D \sim (\sin\alpha)^{1.04}\,, \tag{33}$$

or

$$\mathrm{F_{tgM}} \sim (\sin\alpha)^{2.3}\,. \tag{34}$$

From all the above proportionalities it follows that both the pressure and the drag force and the power input of the impeller significantly increase with increasing angle of inclination of the blades. Eqs (*32*)–(*34*), however, must be taken only as approximations as the proportionalities in the set (*29*) were obtained as the averages over the interval of the angle alpha under consideration by non-linear regression. The function $\varphi_2(c)$ reaches a minimum in the neighbourhood of the angle $\alpha = 45°$. This means that for a higher value of the inclination angle the proportionality (*29b*) cannot be regarded as correct.

Medek[29] reports the dependence of the Euler number for the examined type of paddle impellers (Fig. 3*b*) in the form

$$\mathrm{Eu_M} = 0{\cdot}50\, N^{0.64}\,, \quad [\alpha = 45°,\ \mathrm{Re_M} > 1{\cdot}0\,.\,10^4,\ N \in \langle 2; 6 \rangle]\,. \tag{35}$$

Combining Eqs (*19*), (*30*) and (*35*) one obtains

$$\xi_D \sim (\sin\alpha)^{-0.04}\,. \tag{36}$$

The drag coefficient of the blade is thus practically independent on the number of the blades of the impeller which is in accord with the above conclusion that within the investigated range of N no hydraulic interference of individual blades occurs.

The effect of the width of blade, (h/d), on the value of the power input criterion, $\mathrm{Eu_M}$, has been investigated by O'Kane[21]. From his results it follows

$$\mathrm{Eu_M} = 6{\cdot}20(h/d)\,, \quad [N = 4,\ \alpha = 45°,\ \mathrm{Re_M} > 1{\cdot}0\,.\,10^4,\ (h/d) \in \langle 0{\cdot}10; 0{\cdot}30 \rangle]\,. \tag{37}$$

This is in agreement with the conclusion that within the investigated region of the relative size of the blade both the drag coefficient and the parameter of the velocity profile are independent on this quantity.

Examining the results presented in the theoretical analysis of the hydraulic characteristics of the paddle impellers it should be noted that the form of Eq. (*19*) is a particular case of the generalized form for the power input of the impellers derived by Kvasnička[6] for paddle impellers with flat vertical blades in vessels both with and without baffles. In systems examined in this work, however, the conditions are somewhat more complex. Namely the maximum value of the mean velocity of liquid rotating with the impeller is reached at a point closer to the axis than the radius of the impeller, $d/2$, *i.e.* the condition $0 < c < 1$ is met. At the same time, however, in the subregion of vortex core ($r < cr$) no significant difference exists between the angular velocity of the liquid and the blade, *i.e.* the relative velocity vanishes.

Concluding these considerations it is worth mentioning the theory presented by Van de Vusse[19]. This author views the impellers with flat inclined blades as the so called mixed flow impellers (*i.e.* superimposing radial and axial flow), and not as done in this study as a purely axial impellers. The experimental results of the velocity field in the stream of liquid leaving the region of the rotating blades (rotor region) of the discussed type[9,10,12,14,16], however, are at odds with the cited paper as long as the angle alpha does not exceed $\pi/4$. In contrast, paddle impellers with flat vertical blades or turbine impellers, *e.g.* cit.[30,31], give rise, as has been found, to almost purely radial flow with superimposed tangential component. It may be therefore concluded that within the interval of the angle alpha, $\alpha \in \langle\pi/4; \pi/2\rangle$, both the axial and the radial velocity component will be important in the stream of liquid leaving the region of the rotating impeller. In the available sources of the experimental investigation, however, the mentioned theoretical solution has not been proven although its technical importance is probably interesting and useful.

LIST OF SYMBOLS

A	area, m^2
A	constant
a	exponent
b	width of radial baffle, m
b	exponent
C	constant
c	parameter of the velocity profile defined in Eq. (7)
c	exponent
D	diameter of vessel, m
d	diameter of impeller, m
f	force, N
H	liquid height at rest, m
H_2	height of lower edge of blades over bottom, m
h	width of blade, m
k	geometry parameter of velocity profile defined in Eq. (4)
m_P	number of primary circulations of tracer particle during run
N	number of blades of impeller
n	frequency of revolution of impeller, s^{-1}
P	power input of impeller, W
r	radial coordinate, m
r_c	radial coordinate of maximum velocity within the stream leaving the blades of rotating impeller, m
V	volume of mixed charge, m^3
$\dot{V}$	pumping capacity of mixer, m^3s^{-1}
α	angle of inclination of blades
β	exponent
ε	exponent
r	dynamic viscosity, kg m^{-1} s^{-1}
$\varkappa$	exponent
ϱ	density, kg m^{-3}
σ_z	estimate of standard deviation of quantity z

τ	time, s
$\varphi_1(c)$	function defined in Eq. (*29a*)
$\varphi_2(c)$	function defined in Eq. (*29b*)
$\varphi_3(c)$	function defined in Eq. (*29c*)

Dimensionless Simplexes

Eu_M	Euler (power input) criterion (number) for mixing
F_{tgM}	dimensionless tangential component of force on impeller
K_P	flow rate criterion
Re_M	Reynolds criterion (number) for mixing
ξ_D	drag coefficient of blade

Super- and Subscripts

ax	axial	P	primary circulation
D	drag force	rel	relative
i	summation index	tg	tangential
j	summation index	—	mean value of the given set
M	impeller		

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